JOURNAL OF
COMPUTATIONAL AND
APPLIED MATHEMATICS

ELSEVIER Journal of Computational and Applied Mathematics 83 (1997) 145–146

Erratum

Addenda and Errata to "Linear perturbations of differential or difference operators with polynomials as eigenfunctions" [J. Comput. Appl. Math. 78 (1997) 179–195]

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Some rectifications should be made concerning symmetric linear perturbations.

In Definition 3.6(iii) the condition that if $n = m + 1$ then $q_{nn} \neq 0$ is unnecessary and should be left out. Under the new conditions Theorem 3.7 remains valid and the proof only needs little modification.

In all the subsections of Section 4, "quasi-definite moment functional" should be replaced by "positive-definite moment functional". Heading for Section 4.3 is incorrect and should be replaced by the more interesting:

4.3. Even and odd orthogonal polynomials with a symmetric perturbation

Let $\{P_n(x)\}_{n=0}^{\infty}$ be a system of orthogonal polynomials relative to a positive-definite moment functional σ, such that $P_{2k}(x)$ is an even function and $P_{2k+1}(x)$ is an odd function for all $k = 0, 1, 2, \dots$ and let these polynomials satisfy a differential equation of the form (12). Let ϕ be a symmetric bilinear form of Sobolev type defined by

$$\phi(p,q) = \langle \sigma, pq \rangle + \mu \left[p^{(l)}(-c) q^{(l)}(-c) + p^{(l)}(c) q^{(l)}(c) \right],$$

where $\mu > 0$ and c are real constants, $p^{(l)}(x) = d^l p(x)/dx^l, l \in \{0, 1, 2, \dots\}$, and p and q are any polynomials. Let $P_n^{(l)}(c) \neq 0$ for all $n \geqslant l$. For all $n \geqslant l$ we have $P_n^{(l)}(-c) = (-1)^{n-l} P_n^{(l)}(c)$. The corresponding orthogonal polynomials $\{P_n^{\mu}(x)\}_{n=0}^{\infty}$ can be written as (13) with

$$\begin{aligned} Q_n(x) = {} & \left((-1)^{n-l} K_{n-1}^{(l,l)}(c, -c) + K_{n-1}^{(l,l)}(c, c) \right) P_n(x) \\ & - P_n^{(l)}(c) \left((-1)^{n-l} K_{n-1}^{(0,l)}(x, -c) + K_{n-1}^{(0,l)}(x, c) \right). \end{aligned}$$

PII S0377-0427(97)00116-7

For $0 \leqslant n \leqslant l$, clearly $Q_n(x) \equiv 0$. For $n > k \geqslant l$

$$q_{n,k} = \begin{cases} -\dfrac{2P_n^{(l)}(c)P_k^{(l)}(c)}{\langle \sigma, P_k^2(x) \rangle} \neq 0 & \text{if } n-k \text{ is even,} \\ 0 & \text{if } n-k \text{ is odd.} \end{cases} \tag{37}$$

and

$$q_{n,n} = (-1)^{n-l} K_{n-1}^{(l,l)}(c,-c) + K_{n-1}^{(l,l)}(c,c). \tag{38}$$

It easily follows from (37) and (38) that $\{P_n^{\mu}(x)\}_{n=0}^{\infty}$ is a symmetric linear perturbation of the class l of $\{P_n(x)\}_{n=0}^{\infty}$ (in the new definition, since $q_{l+1,l+1} = 0$) and in a similar way as in the preceding cases we can show that (29) and (30) are satisfied.

The first line of Section 5 "In the papers [4–6, 8, 11]..." can be replaced by "In the papers [4–6, 8, 10, 11]...".